Roll No. ....

94059

B. Sc. (Hons.) Mathematics

5th Semester

Examination – March, 2021

REAL ANALYSIS

Paper : BHM-351

Time : Three Hours] *[Maximum Marks : 60*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. 9 is ***compulsory***. All questions carry equal marks.

SECTION – I

1. (a) If a function f is defined on [0, b], b > 0 by $f(x) = x^3$, then show that f is Riemann integrable on [0, b] and $\int_0^b f dx = \frac{b^4}{4}$.

(b) Prove that every bounded monotonic function is a integrable function.

2. (a) If a function f is continuous on $[a, b]$ and $g(x) = \int_a^x f(t)\,dt$, then prove that g is differentiable on $[a, b]$ and $g' = f$.

(b) Prove that $\frac{1}{\pi} \le \int_0^1 \frac{\sin \pi x}{1+x^2} dx \le \frac{2}{\pi}$.

SECTION – II

3. (a) Show that the integral $\int_0^{\pi/2} \frac{\sin^m x}{x^n} dx$ exists if and only if $n < m + 1$.

(b) Examine the convergence of the improper integral $\int_0^{2a} \frac{1}{(x-a)^2} dx$.

4. (a) Evaluate $\int_0^{\pi} \frac{\log(1 + \alpha \cos x)}{\cos x} dx$ for $|\alpha| < 1$.

(b) If $\int_a^t f\,dx$ is bounded for all $t \ge a$ and g is a bounded and monotonic function for $x \ge a$ tending to '0' as $x \to \infty$, then $\int_a^{\infty} fg\,dx$ is convergent at ∞.

SECTION – III

5. (a) Let (X, d) be a metric space. Prove that a subset of X is closed iff its complement is open.

(b) Prove that in a metric space (X, d), the union of arbitrary collection of open sets is open.

6. (a) If A and B are subsets of metric space (X, d), then prove that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

(b) Prove that every complete metric space is of the second category as a subset of itself.

SECTION – IV

7. (a) Let f be a function of (X, d) into (Y, d^*). Then f is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y.

(b) Prove that a compact subset of a metric space is closed and bounded.

8. (a) Prove that a continuous image of a connected space is connected.

(b) Prove that a metric space (X, d) is compact if and only if it has Bolzano Weierstrass property.

SECTION – V

9. (a) Define norm of a partition of P.

(b) State Abel's test for convergence of improper integral.

(c) Define uniform continuity of a function.

(d) Discuss the convergence of $\int_0^1 \frac{dx}{x^{1/3}(1+x^2)}$.

(e) Define lower and upper sums.

(f) State Baire's category Theorem.